1999: PART A

1) $v(t) = t\sin(t^2)$ for $t \geq 0$

a) $v(1.5) = 1.5\sin(1.5^2) = 1.167$

up because $v(1.5) > 0$

b) $A(t) = v'(t)$
$A(t) = \sin(t^2) + t(2t)\cos(t^2)$
$= \sin(t^2) + 2t^2\cos(t^2)$
$A(1.5) = \sin(1.5^2) + 2(1.5)^2\cos(1.5^2)$
$= -2.049$

v is decreasing since $v'(1.5) < 0$

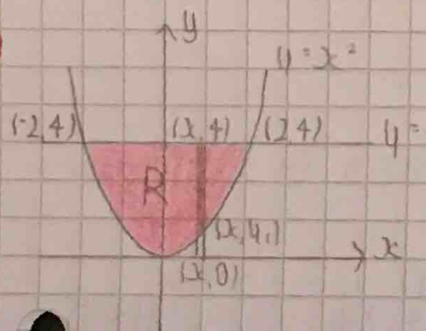
c) $y(t) = \int t\sin t^2 \, dt = -\frac{\cos t^2}{2} + C$
$y(0) = 3$
$-\frac{\cos 0}{2} + C = 3$
$-\frac{1}{2} + C = 3$
$C = 3 + \frac{1}{2}$
$C = \frac{7}{2}$

$y(t) = -\frac{\cos t^2}{2} + \frac{7}{2}$
$y(2) = -\frac{\cos 4}{2} + \frac{7}{2} = 3.827$

d) $t\sin t^2 = 0$ $\qquad 0 \leq t \leq 2$
$t = 0, \ \sin t^2 = 0$ $\qquad 0 \leq t^2 \leq 4$
$t^2 = 0, \pi$
$t = 0, \sqrt{\pi}$

Total Distance $= \int_0^{\sqrt{\pi}} t\sin t^2 \, dt - \int_{\sqrt{\pi}}^{2} t\sin t^2 \, dt$
$= 1.173$ to 3.d.p.

2)



a) Area R $= \int_{-2}^{2} (4 - x^2) \, dx = \frac{32}{3} = 10.667$

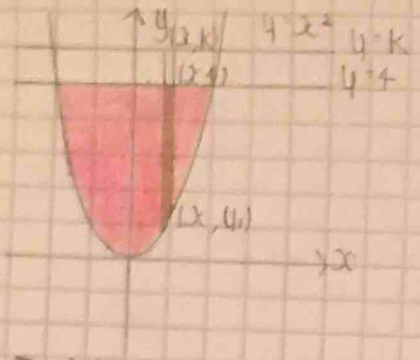
b) $R(x) = 4 - 0 = 4$
$r(x) = y_1 - 0 = x^2$

$\text{Volume} = \pi \int_{-2}^{2} (4^2 - (x^2)^2)\,dx$

$= \pi \int_{-2}^{2} (16 - x^4)\,dx$

$= 160.850$ to 3.d.p.

c)

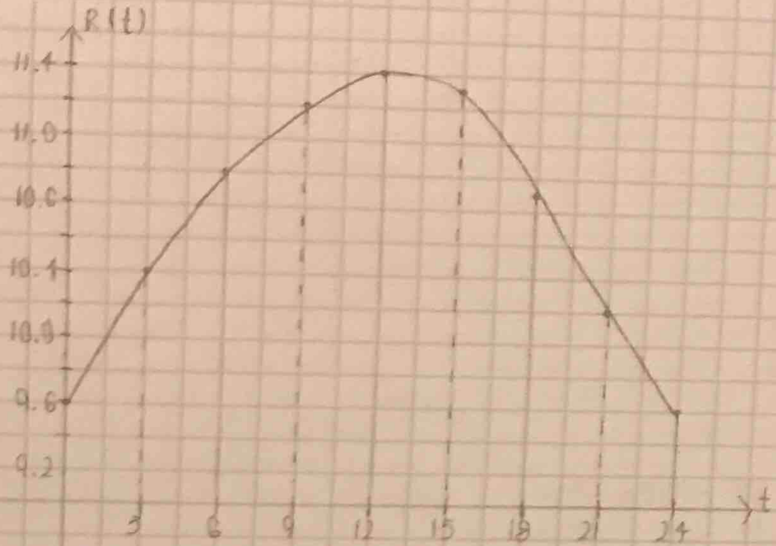


$R(x) = k - y_1 = k - x^2$

$r(x) = k - 4$

$\pi \int_{-2}^{2} ((k - x^2)^2 - (k-4)^2)\,dx = 160.850$

3)



a) $\int_0^{24} R(t)\,dt = \frac{24-0}{4}[R(3) + R(9) + R(15) + R(21)]$
$= 6\,(10.4 + 11.2 + 11.3 + 10.2)$
$= 258.6$ gallons

This is an approximation to the total flow in gallons of water from the pipe in the 24 hour period

b) Since slope is positive $0 < t < 12$ and -ve on $12 < t < 24$ there must be a relative extrema on $0 < t < 24$ $\therefore R'(t) = 0$

Or:
$R(t)$ is continuous because $R(t)$ is differentiable
Since $R(0) = R(24) = 9.6$, the mean value theorem guarentees that there is a t, $0 < t < 24$ such that $R'(t) = 0$

$(t) = \frac{1}{79}(768 + 23t - t^2)$

$$Q(t)_{Av} = \frac{1}{24 - 0}\int_0^{24} Q(t)\,dt$$

$$= \frac{1}{24}\int_0^{24} \frac{1}{79}(768 + 23t - t^2)\,dt$$

$$= 10.785 \text{ gal/hr}$$

4) PART B

4) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$, $f(0) = 2$, $f'(0) = -3$, $f''(0) = 0$

a) $(0, 2)$, $f'(0) = -3$

Equation of tangent:

$y - 2 = -3(x - 0)$
$y - 2 = -3x$
$y = -3x + 2$

b) No. We don't know whether $f''(x)$ changes sign at $x = 0$. We only know $f''(0) = 0$

c) $g(0) = 4 \Rightarrow (0, 4)$

$g'(0) = e^0(3f(0) + 2f'(0))$
$= e^0(3(2) + 2(-3))$
$= 0$

Equation of tangent:

$y - 4 = 0(x - 0)$
$y - 4 = 0$
$y = 4$

d) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$

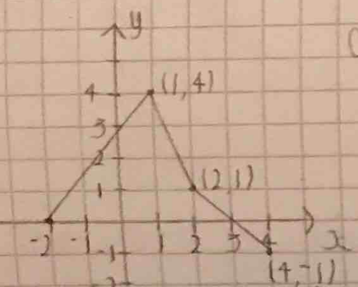
$g''(x) = -2e^{-2x}(3f(x) + 2f'(x)) + e^{-2x}(3f'(x) + 2f''(x))$
$= e^{-2x}(-2(3f(x) + 2f'(x)) + 3f'(x) + 2f''(x))$
$= e^{-2x}(-6f(x) - 4f'(x) + 3f'(x) + 2f''(x))$
$= e^{-2x}(-6f(x) - f'(x) + 2f''(x))$

$g''(0) = e^0(-6(2) - (-3) + 2(0))$
$= 1(-12 + 3)$
$= -9$

$g'(0) = 0$

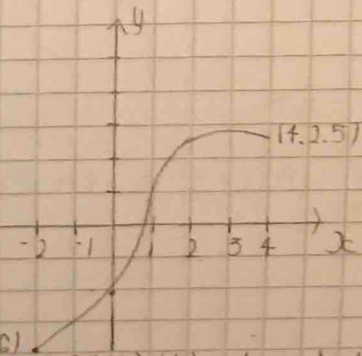
Since $g'(0) = 0$ and $g''(0) < 0$, g has a local maximum at $x = 0$

5)



Graph of f

$g(x) = \int_1^x f(t)\,dt$



a) $g(4) = \int_1^4 f(t)\,dt = \frac{1}{2}(1)(1) + \frac{1}{2}(1)(-1) + \frac{1}{2}(4+1)(1) = \frac{1}{2} - \frac{1}{2} + \frac{5}{2} = \frac{5}{2}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt = -\frac{1}{2}(3)(4) = -\frac{12}{2} = -6$

b) $f(x) = g'(x)$

$g'(1) = f(1) = 4$

c) No relative min therefore absolute min must occur at either endpoint

$g(x)$ is increasing on $[-2, 3]$ and decreasing on $[3, 4]$

$g(-2) = -6$ and $g(4) = \frac{5}{2}$

$\therefore$ absolute minimum value is -6

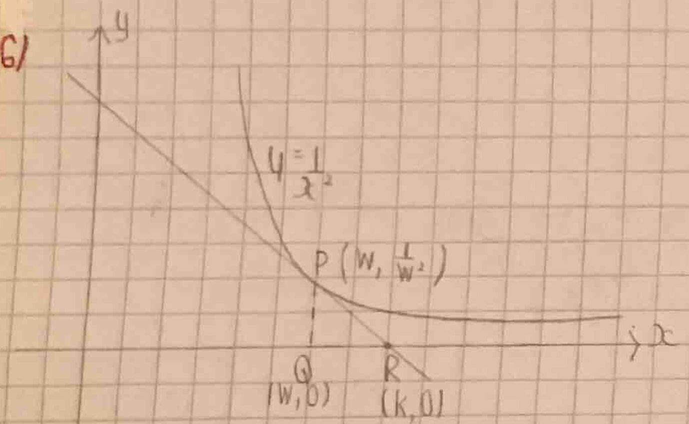
d) only one. $x = 1$ since $g'(x)$ changes from increasing to decreasing at $x = 1$

on $(-2, 1)$, $g''(x) = f'(x) > 0$
on $(1, 2)$, $g''(x) = f'(x) < 0$
on $(2, 4)$, $g''(x) = f'(x) < 0$

* Not defined at $x = 1$, $x = 2$ since slope of $g'(x)$ does not exist

$\therefore (1, g(1))$ is a pt. of inflection and $(2, g(2))$ is not

6)



a) $y = \frac{1}{x^2} = x^{-2}$

when $w = 3$: $P\left(3, \frac{1}{9}\right)$

$y' = -2x^{-3}$
when $x = 3$: $y' = -2(3)^{-3} = \frac{-2}{27}$

Equation of L:
$y - 1/9 = -\frac{2}{27}(x - 3)$

let $y = 0$ for inters. with x axis:
$-\frac{1}{9} = -\frac{2}{27}(x - 3)$
$-3 = -2x + 6$
$2x = 9$
$x = \frac{9}{2}$

$k = \frac{9}{2}$

$y' = -2x^{-3}$

at $\left(w, \frac{1}{w^2}\right)$: $y' = -\frac{2}{w^3}$

Equation of L:

$y - \frac{1}{w^2} = -\frac{2}{w^3}(x - w)$

when $y = 0$

$-\frac{1}{w^2} = -\frac{2}{w^3}(x - w)$

$-\frac{w^3}{w^2} = -2(x - w)$

$-w = -2x + 2w$

$3w = 2x$

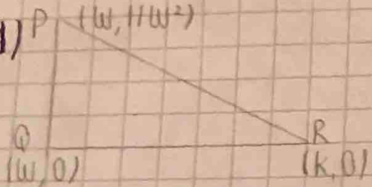
$x = \frac{3}{2}w$

$k = \frac{3}{2}w$

c) $\frac{dk}{dt} = \frac{3}{2}\frac{dw}{dt}$

$\frac{dk}{dt} = \frac{3}{2}(7)$

$\frac{dk}{dt} = \frac{21}{2}$

d) 

$\frac{dw}{dt} = 7$, $w = 5$

$A = \frac{1}{2}(k - w)\left(\frac{1}{w^2}\right)$

$A = \frac{1}{2}\left(\frac{k}{w^2} - \frac{1}{w}\right) = \frac{1}{2}\left(\frac{3}{2}w \cdot \frac{1}{w^2} - \frac{1}{w}\right) = \frac{1}{2}\left(\frac{3}{2w} - \frac{1}{w}\right) = \frac{3}{4w} - \frac{1}{2w} = \frac{1}{4w}$

$A = \frac{1}{4w}$

$\frac{dA}{dt} = -\frac{1}{4w^2}\frac{dw}{dt}$

$\frac{dA}{dt} = -\frac{1}{4(5^2)}(7)$

$= \frac{-7}{100}$

$= -0.07$

The area is decreasing